## Backpaper exam - 2nd semester 2018 B.Math. Hons. 2nd year Algebra IV Instructor : B. Sury

**Q 1.** If L/K is an algebraic extension and  $\alpha, \beta \in L$  have the same minimal polynomial over K, give a clear and complete proof that there is an isomorphism from  $K(\alpha)$  to  $K(\beta)$  which takes  $\alpha$  to  $\beta$  and is identity on K.

**Q** 2. Prove that every finite extension of a finite field is a Galois extension.

**Q** 3. Determine the Galois group of the polynomial  $X^4 - 2$  over  $\mathbb{Q}$ .

**Q** 4. If M is algebraic over L and L is algebraic over K, prove that M is algebraic over K.

**Q 5.** Prove that there exists an irreducible polynomial of any given degree over a finite field.